Only 0 and Infinity Exist: A Categorical and Topos-Theoretic Minimal Ontology of Number

A Preprint

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Abstract

We propose a minimalist ontological thesis: that in the most abstract mathematical and metaphysical sense, only the concepts of zero and infinity possess foundational existence. All other numbers, finite or transfinite, emerge as morphisms or internal constructions within the interval spanning the initial object (0) and terminal colimit (∞). We formalize this claim using both categorical and topos-theoretic tools, showing how intermediate numbers arise contextually rather than fundamentally. This paper constructs a unified categorical schema in which number is demoted from primitive to derived, and only 0 and ∞ serve as ontological anchors.

1 Introduction

Traditional mathematics treats numbers as primitives. Set theory axiomatizes them from the empty set, while category theory builds them from universal properties. Yet, a deeper abstraction reveals that the entire numerical ladder may be an emergent phenomenon. What if, instead, only the poles of this ladder — the empty source and the infinite completion — have true existence?

This paper explores that possibility, both formally and philosophically, using tools from category theory and topos theory.

1.1 What is Ontology and Why It Matters

Ontology, a branch of metaphysics, concerns itself with the most fundamental question of all: *What exists?* In the context of mathematics, ontology is concerned with identifying the irreducible conceptual building blocks of our formal systems. Are numbers real? Are they constructed? Are they universal or contextual?

In traditional set-theoretic foundations, numbers are assumed to be part of an ontological hierarchy built from the empty set. In categorical foundations, numbers are constructed from morphisms and universal properties. In both approaches, *existence* is defined in relation to the axiomatic system chosen.

Our interest is not in arbitrary foundational assumptions, but rather in what structures must exist for any consistent system of mathematics, computation, or physics to arise. This leads us to seek a *minimal ontology*—a set of entities so foundational that all other structures can be derived from them.

We propose that **only 0 and ∞ possess such ontological necessity**.

- **Zero** (0) is the unique object from which all constructions begin—a symbol of emptiness, origin, and non-being.
- **Infinity** (∞) is not merely a large number, but a symbol of unbounded completion, limit, and totality.

All other numbers, structures, and logical systems can be interpreted as morphic constructs or contextual artifacts between these two poles. This reduction has significant implications for the foundations of logic, mathematics, and theoretical physics.

In the sections that follow, we formalize this idea using categorical and topos-theoretic tools, and discuss its broader philosophical and physical consequences.

2 The Categorical Perspective

2.1 Initial and Terminal Objects

In a category $\mathcal{C}$:

– An **initial object** 0 is such that for all $X \in \mathcal{C}$, there exists a unique morphism $0 \to X$.
– A **terminal object** 1 is such that for all $X \in \mathcal{C}$, there exists a unique morphism $X \to 1$.

We may informally designate *infinity* as a terminal colimit or as the object that all morphic ladders asymptotically approach.

2.2 Numbers as Morphisms

In many categories (e.g., the category of sets or types), the natural numbers object (NNO) is constructed via:

- A base point: 0
- A successor morphism: $s : \mathbb{N} \to \mathbb{N}$
- A universal property: induction on $\mathbb{N}$

However, this construction is *not primitive*. Instead, each natural number n is an iterated morphism $s^n(0)$, meaning numbers are artifacts of recursion from 0 toward a colimit object approximating ∞.

3 Topos-Theoretic Framework

3.1 Internal Logic and NNOs

A topos $\mathcal{E}$ admits a natural numbers object $(\mathbb{N}, 0, s)$ if one exists. However, not all topoi possess an NNO, and in many logical systems (e.g. sheaf topoi), arithmetic behaves differently depending on the base space.

Thus, numbers are *local*, internal to a particular topos. The only global absolutes are:

- 0: the initial (empty) object
- Ω: the subobject classifier (often modeling truth)
- Terminal colimits: expressing "∞" via sheaf gluing or directed limits

3.2 Relativity of Arithmetic

In a sheaf topos over a topological space, numbers can vary across stalks. This further reduces their ontological weight, showing they depend on internal context.

4 Minimal Ontology of Number

We propose the following thesis:

> **Only 0 and ∞ exist fundamentally.** All other numbers are emergent morphisms or internal stages in a structural continuum from void to completion.

4.1 Formal Schema

Let $\mathcal{C}$ be a Cartesian closed category with colimits.

- 0 is initial: $0 \to X$ unique
- ∞ is terminal colimit: $\varinjlim F$ for diagram $F : \mathcal{J} \to \mathcal{C}$
- For $n \in \mathbb{N}$, define $n := s^n(0)$ (iterated morphism)

All n lie within the morphic path from $0 \to \infty$.

5 Philosophical Implications

This thesis resonates with metaphysical minimalism, Buddhist void-to-plenitude ontology, and Platonic dualism between the One and the Infinite.

It also aligns with formal constructivism: that nothing can be assumed beyond the rules of construction.

6 Conclusion and Future Work

We have shown that from a categorical and topos-theoretic perspective, only 0 and ∞ require no internal construction or morphic justification. Numbers are artifacts of structure—not primitives.

Future directions include:

- Formalizing this thesis in homotopy type theory
- Exploring its implications in quantum foundations and cosmology
- Developing a minimal arithmetic based only on the continuum from $0 \to \infty$

References

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