Derived Functors and Quantum Error Correction: A Homotopy-Theoretic Approach

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Abstract

This paper introduces a novel approach to quantum error correction (QEC) using derived functors in homotopy theory. By mapping $(\infty, 1)$-spacetime configurations to $(\infty, 1)$-quantum states, our derived functor $H(\infty)$ stabilizes physical content against local deformations and ensures robustness through homotopy-invariant corrections. We demonstrate how this framework addresses the challenges of singularities, anomalies, and topological features in quantum gravity, with practical implications for fault-tolerant quantum computing and dynamic spacetime environments.

1 Introduction

Quantum error correction (QEC) is central to the advancement of fault-tolerant quantum computing. Traditional methods focus on correcting local errors but struggle with nontrivial topological features and singularities inherent in quantum systems influenced by spacetime dynamics. This work proposes a homotopy-theoretic perspective to QEC by utilizing derived functors to emphasize robust invariants.

We define a derived functor:

$$H(\infty) : (\infty, 1)\text{-Spacetime} \to (\infty, 1)\text{-QStates},$$

which enables systematic corrections for cohomological and homotopy-invariant data in quantum states. The implications of this framework include enhanced robustness to dynamic spacetime configurations and improved handling of anomalies and singularities.

2 Homotopy Theory and Derived Functors

Homotopy theory studies deformations of structures up to continuous equivalence, making it a natural candidate for robust quantum error correction. Derived functors extend this by capturing cohomological information in complex systems. By applying $H(\infty)$, we connect geometric configurations with quantum states while preserving topological invariants.

3 Applications to Quantum Error Correction

Our framework offers several improvements over traditional methods:

- **Error Resilience**: Homotopy invariants ensure stability against local deformations.
- **Dynamic Environments**: Topological corrections maintain fidelity in dynamic spacetime configurations.
- **Higher-Dimensional Systems**: The $(\infty, 1)$-category formalism supports higher-dimensional quantum systems.

4 Conclusion

The derived functor $H(\infty)$ introduces a topological perspective to quantum error correction, addressing the limitations of current methods. Future work includes implementing this framework in practical fault-tolerant systems and exploring its implications in quantum gravity.

References

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